COMPRESSED SENSING BASED DENOISING FOR BIOMEDICAL IMAGES

- in theory and in practice -

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Outline

Noise

- perturbation of the signal induced by acquisition device and conditions;
- leads to unwanted variations of color, brightness and/or contrast;
- **biology** and **medicine**: complex acquisition techniques causing many ambiguities.

Example:
microscopic imaging → long exposure time → heating of camera → thermal noise

Denoising

Formalisation: $\mathbf{y} = \mathbf{\Phi x}$

- $\mathbf{x}$ original signal to acquire (aka ground truth)
- $\mathbf{y}$ noisy signal acquired
- $\mathbf{\Phi}$ measurement basis or projection matrix

Denoising:

find optimal approximation $\hat{\mathbf{x}}$ of signal $\mathbf{x}$ from incomplete or inexact measurements $\equiv$ principle of *Compressed Sensing*

Compressed Sensing

Principle of CS: Perfectly reconstruct a signal that is sub-optimally ($\leq 50\%$) sampled, but only if it is sufficiency sparse in some domain $\mathbf{\Psi}$ (sparsity basis, dictionary).

Optimization problem

$$\hat{x} = \arg\min \|\Psi x\|_0 \quad \text{s.t.} \quad y = \Phi x$$

$\mathbf{L_0}$ norm:

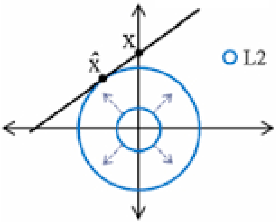
- high computational complexity
- *Matching Pursuit*

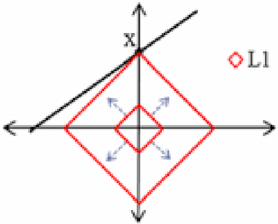
$\mathbf{L_2}$ norm:

- easy to compute
- solution not sparse
- *Least Square Regression*

$\mathbf{L_1}$ norm:

- easier to compute than L_0
- in practice gives same solution as L_0
- *Basis Pursuit*





Constraints relaxation:

- replace L_0 norm with L_1 norm;
- suppose observation is inaccurate:
  $y = \Phi x + b$, where b is additive noise such that $\|b\|_2 \leq \epsilon$.

Optimization problem

$$\hat{x} = \arg\min \|\Psi x\|_1 \quad \text{s.t.} \quad \|\Phi x - y\|_2 \leq \epsilon$$

Property

Most images are sparse in a certain domain:

- space (gradient),
- frequency (wavelets, Fourier)

Constraints relaxation:

- based on the sparsity of signal x in the gradient domain, remove matrix Ψ from problem formulation by replacing L_1 norm with TV norm

$$\|x\|_{TV} = \sum_{p,q} \sqrt{\partial_h x(p,q)2 + \partial_v x(p,q)^2}$$

Optimization
Algorithms

NESTA solves the optimization problem by *accelerated gradient descent* with *back-propagation*:

$$\hat{x} = \arg\min \|x\|_{TV} \quad \text{s.t.} \quad \|\Phi x - y\|_2 \leq \epsilon$$

FISTA minimizes the sum of two convex functions (one smooth and one non-smooth), using *fast gradient projection* it solves another definition of the same optimization problem:

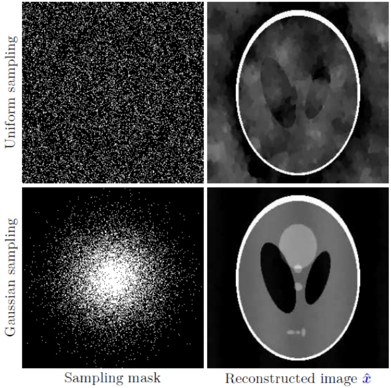
$$\hat{x} = \arg\min \|\Phi x - y\|_2^2 + 2\lambda\|x\|_{TV}$$

where λ (regularization) is a trade-off between fidelity to measurements and noise sensitivity.

- Φ (measurement basis) = **Fourier Transform**,

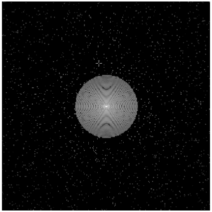
  Note: MRI images are acquired directly in Fourier domain

- most of **spatial energy** is concentrated in the **low frequency** area of the Fourier domain (see Fig.)

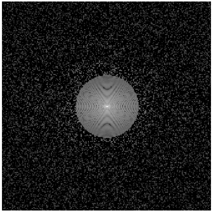


Sampling
Sampling Pattern

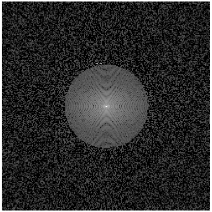
- **partially random** sampling:
  - keep all low frequencies up to a cut-off frequency υ
  - randomly sample high frequencies until sub-sampling rate τ is reached
- 3% - 20% Fourier coefficients $\equiv$ 90% information spatial



$\tau = 10\%,$
$\upsilon = 30\%$



$\tau = 30\%,$
$\upsilon = 30\%$



$\tau = 50\%,$
$\upsilon = 40\%$

Multiple Acquisitions and Reconstructions
Main Idea

Main idea:

- in order to keep a low sampling rate, make multiple sub-optimal acquisitions;

$$y_k = \Phi_k y$$

- merge reconstructions to obtain optimal solution.

$$\hat{x} = fusion(\hat{x_k})$$

Mean:

- average of reconstruction
- smoothen contours
- $\hat{x}_{mean} = \frac{1}{R} \sum_{k=1}^{R} \hat{x}_k$

Variance map:

- standard deviation of reconstructions
- emphasize incoherences (mostly edges)
- as weight matrix (normalized)
- $\sigma_x = \sqrt{\frac{1}{R-1} \sum_{k=1}^{R} (\hat{x}_k - \hat{x}_{mean})^2}$

Fusion operator:

$$\hat{x} = \sigma_x \circ y + (1 - \sigma_x) \circ \hat{x}_{mean}$$

Mean

Variance map

Implementation

input : y, R, τ, υ, λ
output: $\hat{x}$
begin
    for $r \leftarrow 1$ **to** R **do**
        $mask[r] = generateSamplingMask(\tau, \upsilon)$
        $y_i[r] = FFTsampling(y, mask[r])$
        $x_i[r] = FISTAoptimization(y_i[r], \lambda)$
    end
    $\hat{x} = fusion(x_i)$
    return $\hat{x}$
end

Results

Similar results for lower (10%) and higher (40%) sampling rates

Noisy image

Ground truth

$\tau = 10\%$ (PSNR $= 23{,}63$ dB)

$\tau = 40\%$ (PSNR $= 23{,}80$ dB)

Results

Regularization parameter λ very important for high levels of noise

Noisy image

Ground truth

$\lambda = 1$ (PSNR = 14,46 dB)

$\lambda = 10$ (PSNR = 14,77 dB)

Conclusion

Outcome: Denoising Plugin

- uses modern and acclaimed techniques (CS, TV regularization)
- accessible and user friendly (Icy and JVM)
- flexible (multiple adaptable parameters)

Future work:

- add more sampling patterns (Gaussian, fully-random)

References I

W. Meiniel, E. Angelini and J.-C. Olivo-Marin. *Image denoising by adaptive compressed sensing reconstructions and fusions*, in Proc. SPIE 9597, September 2015.

W. Meiniel, Y. Le Montagner, E. Angelini and J. C. Olivo-Marin. *Image denoising by multiple compressed sensing reconstructions*, 2015 IEEE 12th International Symposium on Biomedical Imaging (ISBI), New York, NY, 2015, pp. 1232-1235.

W. Meiniel, Y. Can, C. P. Hendon, J. C. Olivo-Mariri, A. Laine and E. D. Angelini. *Sparsity-based simplification of spectral-domain optical coherence tomography images of cardiac samples*, 2016 IEEE 13th International Symposium on Biomedical Imaging (ISBI), Prague, 2016, pp. 373-376.

Y. LeMontagner. *Algorithmic solutions toward applications of compressed sensing for optical imaging*, Ph.D. thesis, Tlcom ParisTech, 2013.

Stephen Becker, Jrme Bobin, and Emmanuel J. Cands. *NESTA: A Fast and Accurate First-Order Method for Sparse Recovery*, SIAM Journal on Imaging Sciences, 2011.

A. Beck and M. Teboulle. *Fast Gradient-Based Algorithms for Constrained Total Variation Image Denoising and Deblurring Problems*, IEEE Transactions on Image Processing, vol. 18, no. 11, pp. 2419-2434, November 2009.

References II

- Y. Le Montagner, E. Angelini and J. C. Olivo-Marin. *Comparison of reconstruction algorithms in compressed sensing applied to biological imaging*, 2011 IEEE International Symposium on Biomedical Imaging: From Nano to Macro, Chicago, IL, 2011, pp. 105-108.
- Leonid I. Rudin, Stanley Osher, Emad Fatemi. *Nonlinear total variation based noise removal algorithms*, Physica D Nonlinear Phenomena 60, November 1992.
- C. Galambos, J. Kittler, and J. Matas. *Progressive probabilistic hough transform for line detection*, IEEE Computer Society Conference, Los Alamitos, CA, 1999.
- E. J. Cands. *Compressive sampling*, Proceedings of the International Congress of Mathematicians, Madrid, Spain, 2006.
- Hiroyuki Kudo, Taizo Suzuki, Essam A. Rashed. *Image reconstruction for sparse-view CT and interior CTintroduction to compressed sensing and differentiated backprojection*, Quant Imaging Med Surg 2013, pp. 147-161.
- Alan V. Oppenheim, Ronald W. Schafer, and John R. Buck. *Discrete-Time Signal Processing (2nd Ed.)*, Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1999.